Gross FDI co-movement, capital diversity, and welfare gains from financial openness *

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Abstract

Among the G7 economies, gross foreign direct investment (FDI) positions are very large, averaging over 100% of GDP and dwarfing the absolute values of net FDI positions in most countries. Additionally, inward and outward FDI flows exhibit robust positive correlations over the business cycle. To understand these stylized facts on gross FDI flows, we extend the standard international business cycle (IBC) model to allow domestic and foreign ownership of physical capital to be imperfect substitutes. We calibrate the elasticity of substitution between foreign and domestic capital to be 2.12 to match the magnitude of FDI stocks- a value much smaller than the implicitly assumed infinite elasticity in the IBC literature. Given this elasticity we are able to generate a positive co-movement of FDI inflows and outflows consistent with stylized facts. Our results uncover a new source of welfare gains from openness to FDI among otherwise identical developed economies - a *capital diversity* channel, akin to product variety in trade models. The channel is quantitatively important: openness to FDI in the model yields welfare gains equivalent to about a 4-5% increase in life-time consumption, a truly palatable gain to international financial integration.

Keywords: FDI, risk-sharing, international financial integration, international business cycles, Feldstein-Horioka puzzle
JEL codes: E,F

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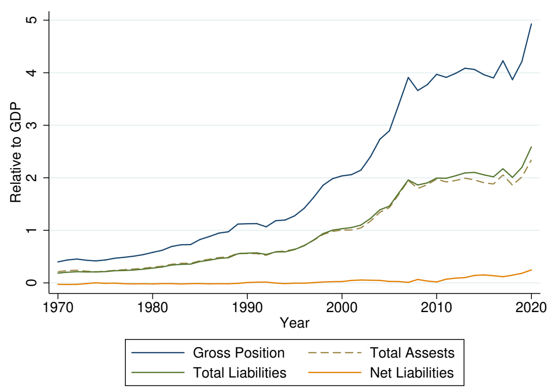
1 Introduction

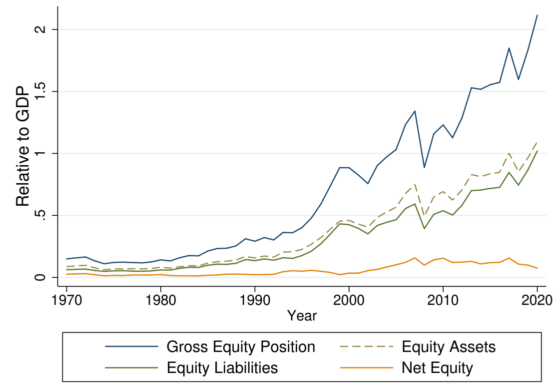
Gross FDI flows are a large and growing share of gross capital flows across the G7, yet we understand little about their dynamics. As Broner et al. (2013) documents, gross capital flows as a whole have increased steadily over the last four decades. These sustained increases in inflows and outflows have had a dramatic impact on the external capital structure for the G7. Figure 1a shows gross international investment positions relative to GDP increasing nearly tenfold. Gross equity positions make up roughly 50% of total gross international investment. Figure 1b suggests that much of the increase in total flows is therefore due to equity, as equity positions have grown nearly 16-fold from the 1980s. Breaking down equity positions into FDI and portfolio equity (PE) investment specifically, similar patterns emerge. Figure 1c plots the evolution of FDI and PE assets collectively, for the G7. According to Figure 1c, while both components of equity have risen dramatically over time, FDI consistently contributes more to gross equity positions than PE. We therefore argue to truly understand gross capital flows one needs to understand gross FDI flows.

The dynamics of gross FDI flows across the G7 are characterized by four stylized facts. The first two hold in the long run while the last two are observed at the business cycle frequency. First, gross FDI positions (assets plus liabilities) of the G7 economies are large, and have more than quadrupled over the last forty years. Second, net FDI (assets less liabilities) is a much smaller fraction of GDP and has changed little over the same period. When aggregating across all countries in the G7, gross FDI as a percentage of GDP increased from 0.5 in 2000 to 1.08 in 2020. Compare this to net FDI as a percentage of GDP, which was 0.048 in 2000 and fell to 0.034 in 2020. Third, examining the business cycle dynamics of FDI flows, we find that gross country-level FDI inflows and outflows are

(a) G7 External Capital Relative to GDP



(b) G7 Equity Position Relative to GDP



(c) FDI Contribution to G7 Gross Equity Position

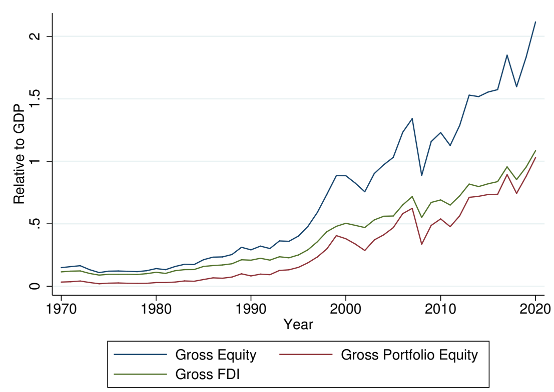


Figure 1

Note:Data is drawn from External Wealth of Nations Database (Lane and Milesi-Ferretti (2018)). In our analysis, we focus primarily on the G7 advanced economics – U.S., Germany, France, Great Britain, Italy, Canada, and Japan. We use this data to focus on the stock of international capital. External financial assets (liabilities) are claims by domestic residents (nonresidents) on nonresidents (residents).

positively correlated over the business cycle, exhibiting a similar behavior as total capital inflows and outflows previously documented by Broner et al. (2013). And finally both are procyclical. On average, the unconditional correlation between inward and outward FDI flows is 0.470, with all 7 countries having positive correlations ranging from 0.165 (Japan) to 0.643 (Italy).

Given the growth in gross FDI positions and consistent patterns in gross flows across the business cycle, it is surprising that the vast majority of the theoretical IBC literature is silent about the mechanisms that are driving the dynamics of gross FDI.[1] To fill this gap, we extend the classic framework of Backus et al. (1992) (BKK henceforth) to account for (i) large gross FDI flows (ii) small net FDI flows, (iii) a positive co-movement of FDI inflows and outflows, and (iv) the pro-cyclical nature of both inflows and outflows of FDI. These stylized facts suggest that, at the aggregate level, the domestic and foreign ownership of physical capital stock are not perfect substitutes. We thus proceed by relaxing the assumption of infinite elasticity of substitution of foreign and domestic capital that is implicit in all BKK-based models. This straightforward modification allows our model to have well-defined gross FDI stocks and flows.

Our framework uses the long run moments on gross and net FDI (stylized facts (i) and (ii)) to pin down the aggregate elasticity of substitution between foreign and domestic capital, and then we compare the resulting business cycle moments from the model to data (stylized facts (iii) and (iv)). With this strategy, we calibrate the elasticity of substitution to be 2.12 to match the magnitude of FDI stocks- a value much smaller than the implicitly assumed infinite elasticity in the IBC

[1] Bai (2013) notes the absence in theoretical work on gross capital flows more generally. The three most famous puzzles in international capital flows (Feldstein-Horioka, Lucas puzzle, and the Allocation Puzzle) are all focused on net flows. The literature on capital flows in emerging markets similarly focuses on net flows and net stocks. Kaminsky et al. (2005) look at the cyclical behavior of total net flows, while Smith and Valderrama (2009) examine the difference between net FDI compared to net debt positions.

literature. Given this elasticity we are able to generate a positive co-movement of FDI inflows and outflows consistent with stylized facts. We also find that as the elasticity of substitution approaches infinity (the implicit value in the BKK framework and the papers that followed), the co-movement between FDI inflows and outflows approaches negative one, counterfactual to the data.

Our results uncover the existence of a new source of welfare gains from financial integration, which we dub *capital diversity.* Our estimate of the elasticity of substitution between domestic and foreign ownership of capital stock in the aggregate production function implies that the welfare gains from openness to FDI via the capital diversity channel alone are equivalent to at least a 5% increase in life-time consumption and could be as high as 10-15%, far exceeding the elusive gains reported by Gourinchas and Jeanne (2006). In their calibration, non-OECD countries benefit from international financial integration by around 1% while OECD countries do not benefit at all. Our simple departure from standard IBC models allows for diversity in aggregate capital stock to raise aggregate productivity, a source of welfare gains for OECD countries absent in Gourinchas and Jeanne (2006).

Capital diversity implies ownership matters. Foreign and domestic owners use capital differently, leading the capital stocks to be imperfect substitutes. Our notion of *capital diversity* is related to studies by McGrattan and Prescott (2009) and by Hoxha et al. (2013). McGrattan and Prescott (2009) introduces a concept of intangible technology capital, which is a non-rival capital good that can be used in multiple locations, generating large welfare gains from openness to FDI. Our model in contrast assumes the intangible capital is embedded in the capital goods and therefore cannot be used across locations. Hoxha et al. (2013) looks at welfare effects of financial openness in developing countries when capital goods are imperfect substitutes. They find (as do we) that a low elasticity of substitution translates into large welfare gains. They consider different values of the elasticity of substitution, based on a number of micro estimates (Goolsbee, 2004; Chun and Mun, 2006), whereas our focus is on the aggregate elasticity of substitution, which is well-identified in our model.

There are a variety of micro-oriented interpretations of an aggregate finite elasticity of substitution between foreign and domestic capital ownership. Mechanically, the *capital diversity* channel acts in a manner very similar to the love for variety in trade models (Armington (1969) and Krugman (1979)). For example, Petrosky-Nadeau (2011) model domestic and foreign firms facing frictions in their search for productive opportunities in both domestic and foreign locations, which would generate a finite elasticity in an aggregate CES specification. Antràs et al. (2009) consider international investment decisions in the face of incomplete contracting and managerial incentives, where capital ownership brings particularly useful monitoring capabilities that ensure optimal deployment of technologies. In broad strokes, micro explanations for our aggregate estimates fall into either within-firm, across-firm within-industry, or across-industry mechanisms.[2]

Our steady-state welfare results are a lower bound on the full effects of the capital diversity channel. Introducing interactions between capital diversity and risk sharing (Mendoza and Tesar, 1998; van Wincoop, 1999) or allowing for faster convergence of capital stock to a new steady-state (Dell'Ariccia et al., 1998; Gourinchas and Jeanne, 2006) would further increase the tangible gains from international financial liberalization. For example, in our model, once the foreign capital is installed, both the stock of FDI and adjustment to that stock (FDI flows) serve as a(n imperfect)

[2]See Mcquoid et al. (2023) for firm level estimations of these effects using Census data.

risk-sharing mechanism. When one country receives a positive productivity shock, the returns to domestic and foreign owned capital both go up, thereby spreading out the income shocks across countries. Furthermore, the country can adjust consumption inter-temporally through investing domestically and in the foreign economy through FDI outflows.

In addition, our model generates relatively high correlations between savings and investment rates. Absent the ability to trade a non-contingent bond and with less than perfect substitution between domestic and foreign ownership of capital, international financial markets are incomplete. This has two important consequences. First, it emphasizes the importance of the capital diversity channel as a source of welfare gains from financial openness. The overall gains are large even though the gains from risk-sharing are reduced (because of market incompleteness). Second, it helps us account for an important puzzle in open economy macroeconomics - high correlations between domestic savings and investment (the FH puzzle).

This is the first work to present an empirically identifiable framework that accounts for large gross FDI ownership and generates meaningful predictions about gross FDI inflows and outflows.[3] The international business cycle literature has paid careful attention to international trade in assets, but most of that attention focused on the role of market incompleteness (Baxter and Crucini, 1995; Heathcote and Perri, 2002), the extent of international diversification of passive, short-term portfolio holdings (Baxter and Jermann, 1997; Heathcote and Perri, 2013), or choices between holdings safe vs. risky assets (Devereux and Sutherland, 2009; Davis and van Wincoop, 2022).

The rest of the paper proceeds as follows. Section 2 describes stylized facts regarding gross

[3] Davis and van Wincoop (2022) recently developed a theory of gross capital flows that would account for a drop in gross capital flows during a global financial crisis. Their focus is on the short-term portfolio allocation of safe and risky assets, rather than on the direct investment flows with an active participation in the production process.

equity positions and flows for the G7. Section 3 provides an explanation of the theoretical model. Section 4 explains our calibration and estimation procedure and our results. Section 5 suggests some of the policy implications from *capital diversity* and provides concluding comments.

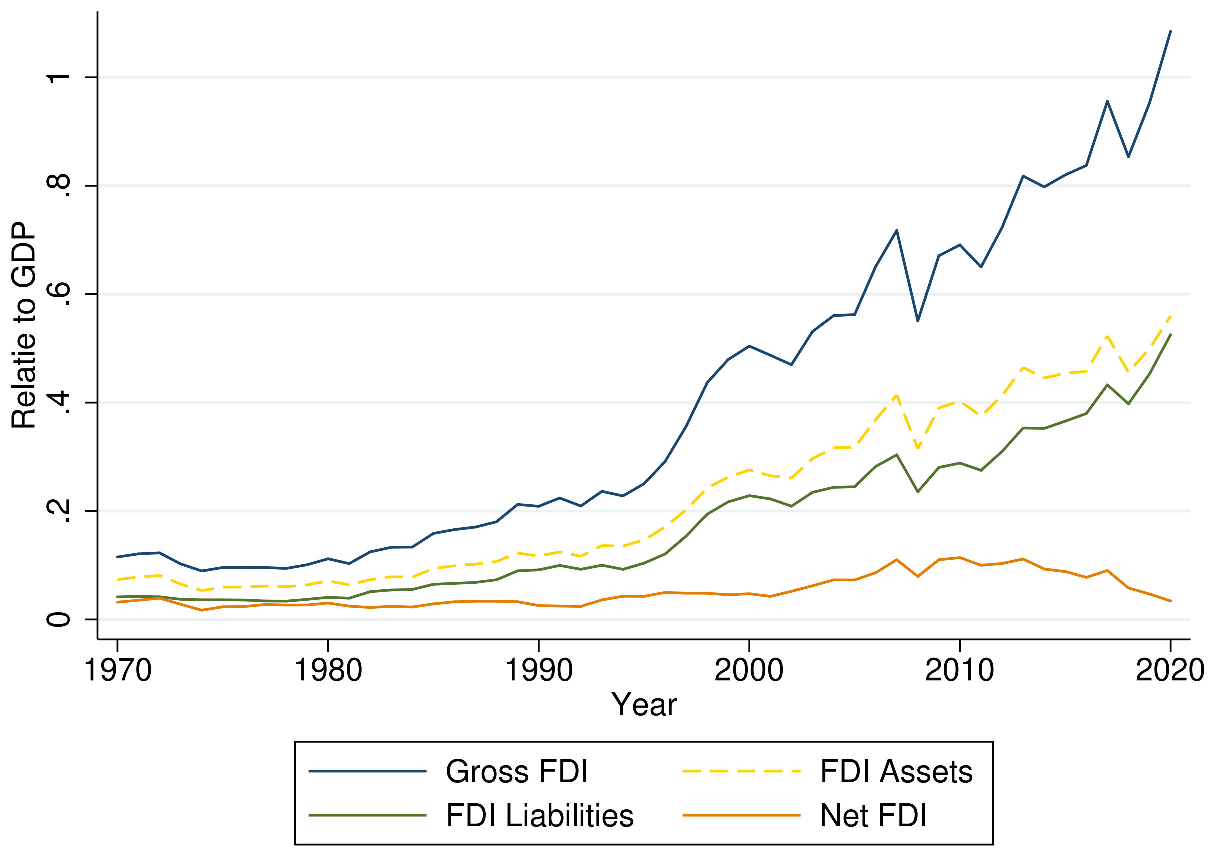
2 Gross Capital Flows over the business cycle: stylized facts

In this section, we focus on documenting four stylized facts among G7 countries that motivate and discipline our methodological approach below. The first two stylized facts center on long run dynamics, while the second two focus on business cycle behaviors, namely: (i) gross FDI positions are large and growing, (ii) net FDI positions are small and consistent over time, (iii) inward and outward FDI flows are positively correlated, and (iv) inward and outward FDI are both procyclical. Taken together, these facts suggest that gross FDI flows are important, and their patterns provide useful moments in the data to better understand international financial integration.

The growth in gross FDI stocks over the last few decades has been remarkable, as can be seen in Figure 2. Gross FDI as a percentage of GDP has increased over tenfold since 1970. At the same time, net FDI in 2020 was roughly the same as it was in 1970 (relative to GDP). These two observations follow from the large increases in FDI assets and FDI liabilities, but since the increases were similar, there was little effect on the net between them. A myopic focus on net FDI without considering the context of growing gross FDI may lead to an undervaluation of the impact of FDI.

With an eye towards understanding the importance of the *capital diversity channel*, we center our attention on changes in gross equity and FDI flows over the long run and across the business cycle. Using data from the IMF's Balance of Payments database for the G7 advanced economies, Table 1 provides summary statistics overall and for each decade for gross equity flows. Each variable

Figure 2: G7 FDI Stocks Relative to GDP



is measured relative to trend GDP. Gross FDI, PE, and gross FDI plus PE follow similar patterns over time. Gross FDI has a median value of 2.7% of trend GDP over the whole sample, which ranges from a low of 0.9% in the 1970s to 4.7% in the 2000s. Gross FDI flows relative to trend GDP increased by a factor of 4 compared to the 1970s. Gross PE flows averaged 3.6% over the whole sample, and grew by a factor of 10 between the 1970s and the 2020s. The combination of gross FDI and PE flows made up nearly 10% of trend GDP from 2010-2020.

Table 1: FDI and Portfolio flows relative to GDP

1970-2020		Median	Mean	Std Dev	
Gross FDI		0.027	0.036	0.038	
Gross PE		0.036	0.050	0.054	
Gross FDI+PE		0.067	0.085	0.079	
Medians	1970s	1980s	1990s	2000s	2010s
Gross FDI	0.009	0.011	0.027	0.047	0.040
Gross PE	0.005	0.019	0.047	0.078	0.052
Gross FDI+PE	0.016	0.031	0.081	0.131	0.087
Standard Deviations	1970s	1980s	1990s	2000s	2010s
Gross FDI	0.012	0.016	0.039	0.055	0.027
Gross PE	0.009	0.023	0.045	0.072	0.042
Gross FDI+PE	0.016	0.036	0.068	0.104	0.048

Note: FDI and PE Flows are measured relative to trend GDP. Source: IMF's Balance of Payments database.

Table 1 provides a similar breakdown of volatility for each type of capital flow. Gross PE flows are more volatile than gross FDI flows, and they are positively correlated, resulting in a larger standard deviation for the aggregate measure of FDI plus PE. When broken out by decades, gross FDI flows were most volatile in the 2000s, and while volatility declined in the 2010s, gross FDI volatility has more than doubled over the last five decades. Similar patterns emerge for gross

Table 2: FDI flows relative to Domestic Investment

		All Years	1970s	1980s	1990s	2000s	2010s
Gross FDI Flows	Median	0.122	0.034	0.047	0.123	0.216	0.192
	Mean	0.168	0.062	0.076	0.181	0.298	0.190
	Std Dev	0.194	0.048	0.065	0.202	0.287	0.134
Net FDI Flows	Median	-0.019	0.000	-0.009	-0.023	-0.042	-0.034
	Mean	-0.024	0.002	-0.014	-0.037	-0.040	-0.021
	Std Dev	0.081	0.028	0.026	0.064	0.114	0.107
FDI Inflows	Median	0.052	0.018	0.022	0.055	0.095	0.078
	Mean	0.072	0.032	0.031	0.072	0.129	0.084
	Std Dev	0.092	0.030	0.030	0.079	0.137	0.088
FDI Outflows	Median	0.063	0.021	0.032	0.065	0.134	0.115
	Mean	0.096	0.030	0.045	0.109	0.169	0.106
	Std Dev	0.117	0.025	0.040	0.127	0.170	0.083

Note: FDI flows are measured relative to gross capital formation, drawn from the World Development Indicators. Net FDI inflows are measured as inflows minus outflows. Gross FDI is measured as inflows plus outflows.

portfolio flows.

Table 2 looks at gross FDI flows relative to domestic investment. For all G7 countries across all years, the median value of gross FDI flows to domestic investment is 0.122, with a standard deviation of 0.194. When dis-aggregated across decades, we see a rising influence of gross FDI relative to domestic investment from the 1970s to the 2000s, increasing roughly 7-fold between the 1970s and 2000s. If we look at averages instead of medians, we see a 5-fold increase over this time period. The standard deviation goes from 0.048 in the 1970s to 0.287 in the 2000s.

The following decade shows a break in this upward trend, although the median is still about 50% higher in the 2010s when compared to the 1990s. All told, over the five decades under consideration, gross FDI flows as a percent of domestic investment have tripled in magnitude, while the standard deviation has more than doubled. Taken over the entire sweep of the sample, it is clear that gross FDI is playing an increasingly important role as a share of domestic investment. Such dynamics are not captured when net FDI flows are used in the place of gross FDI flows (see Table 2).

Having documented key features of long run gross FDI dynamics, we next turn to exploring gross FDI over the business cycle. While FDI outflows, inflows, and gross flows are all strongly pro-cyclical, net FDI flows have no clear trend over the business cycle. Table 3 reports correlations between ln (real GDP) and four measures of FDI flows (all as a percent of GDP). For all variables, we have taken the cyclical component after using an HP filter. We find that the average correlation for gross FDI flows and real GDP is 0.4 in our sample, with a median of 0.32. The highest correlation comes from Canada (0.66), while Germany has the lowest at 0.15. For net FDI flows, 5 of our 7 countries have negative correlations, with an average correlation of -0.06. Thus, while net FDI flows show no distinct patterns over the business cycle, gross FDI flows are strongly pro-cyclical.

Perhaps most importantly for our approach below, we document a strong positive correlation between FDI inflows and outflows for our panel of G7 countries in Table 4. Country by country for both FDI inflows and outflows, we de-mean and standardize each series. We then estimate the impact of FDI outflows on inflows (and separately inflows on outflows), including country-specific time trends as well as year dummies. We consider 1970-2010 and 1970-2020 to evaluate whether the decade following the global financial crisis altered the relationship between inflows and outflows (having seen already that magnitudes of flows declined during this period). Table 4 show inflows and outflows of FDI are highly positively correlated, and these estimated relationships are robust to sample selection and additional controls. Table 5 reports the correlations between FDI inflows and

Table 3: Correlations between measures of FDI and Real GDP

	ρ (Gross FDI, y)	ρ (FDI In, y)	ρ (FDI Out, y)	ρ (Net FDI, y)
Canada	0.663	0.663	0.437	0.454
Germany	0.153	0.093	0.209	-0.018
France	0.538	0.581	0.432	-0.228
United Kingdom	0.289	0.139	0.350	-0.300
Italy	0.513	0.440	0.489	-0.093
Japan	0.320	-0.187	0.458	-0.528
United States	0.244	0.409	0.041	0.317
Average	0.389	0.305	0.345	-0.057
Median	0.320	0.409	0.432	-0.093
Std Dev	0.184	0.302	0.164	0.345

Note: y is measured as the cyclical component using an HP filter on the natural log of real GDP. FDI measures are the cyclical component using an HP filer on each FDI series as a percent of GDP.

FDI outflows for each country separately. A clear pattern emerges that FDI inflows and outflows are highly positively correlated, with a high of 0.64 (Italy) to a low of 0.165 (Japan). On average, the correlation of FDI inflows and FDI outflows is 0.467.

Having documented important features of gross and net FDI flows over time, and over the business cycle, we next turn to developing a theoretical framework that can accommodate these stylized facts.

3 Model

To understand the cyclical patterns in FDI evident in the data, we augment the classic international business cycle framework of BKK. There are two countries in the model: A and B. The GDP in each country is produced using capital and labor, and is then used for either consumption or investment purposes. The consumption and investment goods in both countries are perfect substitutes.

Table 4: Correlations between FDI Inflows and FDI Outflows

	(1) FDI In	(2) FDI Out	(3) FDI In	(4) FDI Out	(5) FDI In	(6) FDI Out	(7) FDI In	(8) FDI Out
FDI Outflows	0.761**** (0.0724)		0.317*** (0.106)		0.707**** (0.0525)		0.455**** (0.0702)	
FDI Inflows		0.733**** (0.0835)		0.288** (0.118)		0.705**** (0.0698)		0.470**** (0.0874)
Standard Errors	Robust	Robust	Robust	Robust	Robust	Robust	Robust	Robust
Country-Specific Trend	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year Dummies	No	No	Yes	Yes	No	No	Yes	Yes
Time Period	1970-2010	1970-2010	1970-2010	1970-2010	1970-2020	1970-2020	1970-2020	1970-2020
Observations	248	248	248	248	332	332	332	332

Note: FDI inflows and outflows are recorded as positive values.

Table 5: Correlations between FDI Inflows and Outflows for each G7 country

	ρ (FDI in, FDI out)
Canada	0.508
Germany	0.439
France	0.449
United Kingdom	0.599
Italy	0.643
Japan	0.165
United States	0.467
Average	0.467
Median	0.467
Std Dev	0.154

Note: based on residuals from an HP filter.

Technology and resource constraints The original BKK framework does not have a well defined concept of gross FDI stocks and flows. We augment the model so that the notion of capital ownership is meaningful and it makes a difference how much of the capital located physically in country i is owned by domestic vs. foreign residents. We do so by defining a concept of effective capital stock, $\tilde{K}$, as follows:

$$\tilde{K}_i = \left[\omega k_i^{\frac{\theta-1}{\theta}} + (1-\omega) k_i^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \qquad i = A, B \tag{3.1}$$

where k_i is owned by domestic residents, k_i^* is owned by foreign investors, ω is the home bias, and θ is the elasticity of substitution between domestic and foreign ownership. This is our key innovation, and the key difference between the classic BKK framework and ours. In the standard BKK framework it does not matter who owns capital stock, which corresponds to $\theta = \infty$. In that framework gross FDI flows are not well defined, and only net flows matter.

The notion of effective capital stock in (3.1) hints at additional sources of welfare gains from financial integration. In the standard BKK framework, the only source of those gains is insurance against country-specific shocks, i.e. risk-sharing. When the elasticity of substitution between domestic and foreign capital is finite, the increase in the effective capital stock that follows financial integration will be larger than the increase in the sum of its components. Hence, financial integration has a potential for increasing steady-state levels of output and consumption.

There are at least two ways in which the concept of effective capital stock can be interpreted. One is similar to the idea of intangible capital in McGrattan and Prescott (2009). Another interpretation is countries benefiting from having comparative advantage in different sectors, resulting from accumulated know-how - the Swiss may own part of German watch-making sector, while Germans may own part of the Swiss automobile industry.

In each country $i = A, B$, the effective capital stock, along with labor, is used to produce a final consumption and investment good, using a Cobb-Douglas production function:

$$Y_i\left(s^t\right) = e^{z_i\left(s^t\right)} \cdot \tilde{K}_i\left(s^{t-1}\right)^{\alpha} L_i\left(s^t\right)^{1-\alpha}, \qquad i = A, B \tag{3.2}$$

where s^t is the exogenous state of the world encompassing the history of all past shocks, $s^t = (s_1, s_2, ..., s_t)$, z is the logarithm of total factor productivity, and L is labor input.

The capital stocks located in country $i = A, B$ and owned by either Home or Foreign residents evolve over time as follows:

$$k_i\left(s^t\right) = (1-\delta)k_i\left(s^{t-1}\right) + x_i\left(s^t\right) - \frac{\phi_D}{2}\left(\frac{k_i\left(s^t\right)}{k_i\left(s^{t-1}\right)} - 1\right)^2 \cdot k_i\left(s^{t-1}\right) \tag{3.3}$$

$$k_i^*\left(s^t\right) = (1-\delta)k_i^*\left(s^{t-1}\right) + x_i^*\left(s^t\right) - \frac{\phi_F}{2}\left(\frac{k_i^*\left(s^t\right)}{k_i^*\left(s^{t-1}\right)} - 1\right)^2 \cdot k_i^*\left(s^{t-1}\right) \tag{3.4}$$

where x_i and x_i^* are purchases of country i capital goods made by domestic and foreign residents, respectively. It's important to notice that nothing restricts either x_i or x_i^* from being negative. The parameters ϕ_D and ϕ_F measure the size of the capital adjustment costs, potentially differing for capital located domestically or in a foreign country.

Equations (3.1), (3.3), and (3.4) create a situation where we can differentiate between the domestic and foreign ownership. Having separate adjustment costs for k_i and k_i^* immediately differentiates between domestic and foreign ownership away from the steady-state, even if $\theta = \infty$. When $\theta = \infty$, however, the steady-state levels of k_i and k_i^* are indeterminate.

The global resource constraint is given by:

$$C_A\left(s^t\right)+C_B\left(s^t\right)+x_A\left(s^t\right)+x_A^*\left(s^t\right)+x_B\left(s^t\right)+x_B^*\left(s^t\right)=Y_A\left(s^t\right)+Y_B\left(s^t\right) \tag{3.5}$$

Preferences The expected life-time utility of a stand-in household in each country is given by:

$$\sum_{t=1}^{\infty}\beta^t\left[\sum_{s^t}\pi\left(s^t\right)\psi\left(s^t\right)U\left(c\left(s^t\right),\ell\left(s^t\right)\right)\right],$$

with a similar specification in country B, where c is consumption, ℓ is hours worked, and ψ represents the inter-temporal preference shock, introduced to ensure that the model captures the relative volatility as well as the cross-country correlation of consumption expenditures.

Stochastic shocks There are two stochastic shocks in each country - a shock to the total factor productivity, z, and the inter-temporal preference shock, ψ. We assume the following stochastic process for the two shocks in each country $i=A,B$:

$$z_{i,t}=\rho_z z_{i,t-1}+\epsilon^z_{i,t}, \qquad i=A,B \tag{3.6}$$

$$\ln\psi_{i,t}=\rho_\psi\ln\psi_{i,t-1}+\epsilon^\psi_{i,t}, \qquad i=A,B \tag{3.7}$$

The four shocks have a joint normal distribution, and are potentially correlated across countries:

$$\left(\epsilon^z_{A,t},\epsilon^z_{B,t},\epsilon^\psi_{A,t},\epsilon^\psi_{B,t}\right)^T\sim N\left(0,\Sigma\right),$$

with the variance-covariance matrix assumed to take the following form:

$$\Sigma = \begin{bmatrix} \sigma_z^2 & \sigma_z^2 \cdot \rho_{z,z^*} & 0 & 0 \\ \sigma_z^2 \cdot \rho_{z,z^*} & \sigma_z^2 & 0 & 0 \\ 0 & 0 & \sigma_\psi^2 & \sigma_\psi^2 \cdot \rho_{\psi,\psi^*} \\ 0 & 0 & \sigma_\psi^2 \cdot \rho_{\psi,\psi^*} & \sigma_\psi^2 \end{bmatrix}$$

3.1 Planner's problem

The planner's problem (assuming equal welfare weights across countries) can be set up by treating k_A and k_A^* (as well as k_B and k_B^*) as distinct capital goods. The planner solves the following problem:

$$\max \sum_{i \in \{A,B\}} \sum_{t=1}^{\infty} \beta^t \left[\sum_{s^t} \pi\left(s^t\right) \psi_i\left(s^t\right) \cdot U\left(C_i\left(s^t\right), L_i\left(s^t\right)\right) \right]$$

subject to (3.1) - (3.7).

3.2 Competitive Equilibrium

In the decentralized economy, we consider two types of assets that can be traded internationally. First are the ownerships of the physical capital stocks located in a different country. Second are claims to future consumption, i.e. a non-contingent, international bond (debt). We add capital controls to both types of assets (separate for each) so that we can consider different types and degrees of financial integration.

Households' utility maximization A stand-in household in country A is endowed with a unit of labor and solves:

$$\max \sum_{t=1}^{\infty} \beta^t \left[\sum_{s^t} \pi\left(s^t\right) \psi\left(s^t\right) U\left(c(s^t), \ell\left(s^t\right)\right) \right]$$

subject to:

$$c(s^t) + x_A\left(s^t\right) + x_B^*\left(s^t\right) \leq w(s^t)\ell\left(s^t\right) + r_A\left(s^t\right) k_A\left(s^{t-1}\right) + \left(1-\kappa^F\right) r_B^*\left(s^t\right) k_B^*\left(s^{t-1}\right) + q\left(s^t\right) d\left(s^t\right) - d\left(s^{t-1}\right) - \frac{\kappa^D}{2\cdot\left(1-\kappa^D\right)} d\left(s^t\right)^2 + T\left(s^t\right) \tag{3.8}$$

$$k_A\left(s^t\right) - (1-\delta)k_A\left(s^{t-1}\right) \leq x_A\left(s^t\right) - \frac{\phi_D}{2}\left(\frac{k_A\left(s^t\right)}{k_A\left(s^{t-1}\right)} - 1\right)^2 \cdot k_A\left(s^{t-1}\right) \tag{3.9}$$

$$k_B^*\left(s^t\right) - (1-\delta)k_B^*\left(s^{t-1}\right) \leq x_B^*\left(s^t\right) - \frac{\phi_F}{2}\left(\frac{k_B^*\left(s^t\right)}{k_B^*\left(s^{t-1}\right)} - 1\right)^2 \cdot k_B^*\left(s^{t-1}\right) \tag{3.10}$$

where w is the real wage, $d(s^t)$ is the debt issued in state s^t, q is the price of newly issued debt, and T are lump-sum transfers, taken as given by the household, and equal to:

$$T\left(s^t\right) = \kappa^F \cdot r_A^* k_A^*\left(s^t\right) + \frac{\kappa^D}{2\cdot\left(1-\kappa^D\right)} d\left(s^t\right)^2$$

We rebate the portfolio adjustment cost back to the household so that any increase in κ^D only captures the distortionary effect of incomplete markets. The household takes as given all prices as well as the aggregate allocations. The stars on r_B^* and k_B^* reflect the fact that from country B perspective, household A is a foreign household. The problem for the household in country B is similar.

Firms' profit maximization In country A, a representative firm maximizes profits by solving:

$$\max_{\tilde{K},L,k_A,k_A^*} e^z \cdot \tilde{K}^\alpha L^{1-\alpha} - wL - r_A k_A - r_A^* k_A^* \qquad \text{subject to (3.1)}$$

The profit maximization problem in country B is similar. Since the problem is static, we dropped the explicit notation that all prices and allocations are functions of the aggregate state s^t.

Capital controls and financial integration The two parameters, κ^F and κ^D capture various degrees of financial integration. The first one, κ^F, is a tax imposed on return to capital earned by foreign owners, that is then lump-sum rebated to domestic residents. Specifically, if the rental rate on foreign owned capital in country A is r_A^*, the foreign owner receives a payment of $\left(1 - \kappa^F\right) r_A^* k_A^*$ and the amount $\kappa r_A^* k_A^*$ is rebated to the stand-in household in country A (similar tax and transfer is taking place in country B). The second one, κ^D is the cost associated with ending a period with a non-zero amount of foreign debt.

The two parameters are restricted to be between 0 (financial integration) and 1 (autarky). When $\kappa^F = \kappa^D = 0$, international financial markets are complete, and the competitive equilibrium allocations are the same as the solution to the planner's problem (see Theorem 3.2 in the next section). When $\kappa^F = 0$ but $\kappa^D = 1$, there is freedom to buy, own, and sell physical capital stock located in a different country, but the cost of issuing any amount of non-contingent debt is prohibitively high. The markets are incomplete, but there is still international trade in assets in the form of FDI and, in general, $NX \neq 0$.[4] When $\kappa^F = 1$ but $\kappa^D = 0$, any return from foreign-owned

[4] This intermediate (between autarky and complete markets) level of financial integration is different from the one typically considered in the international business cycle literature (Heathcote and Perri, 2002; Corsetti et al., 2008; Rothert, 2020). In those papers, the intermediate level of integration relies on consumption smoothing via the international bond. The case of $\kappa^F = 0$ and $\kappa^D = 1$ shuts down the risk-sharing via the non-contingent bond, while still allowing for international holdings of foreign assets.

capital is confiscated and only non-contingent debt can be issued. Finally, when $\kappa^F = \kappa^D = 1$, we have a financial autarky.

Definition 3.1 (Competitive Equilibrium)**.** A competitive equilibrium consists of price and allocation functions: $\left[C_i(s^t), L_i(s^t), \tilde{K}_i(s^t), k_i^j(s^t), Y_i(s^t), r_i^j(s^t), w_i(s^t), x_i^j(s^t), q(s^t), T_i(s^t)\right]_{i,j=A,B}$, such that, given prices, allocations solve the utility and maximization problems, and all markets clear.

3.3 Characterization

We provide partial characterization of the model to facilitate the explanation of our main results later on. We start with the relationship between the allocations that solve the planner's problem and the allocations in the competitive equilibrium, summarized in Theorem 3.2.

Theorem 3.2. *Let $\hat{Z}$ be the allocation that solves the Social Planner's problem and let $\tilde{Z}$ be the allocation in the Competitive Equilibrium. Then $\hat{Z} = \tilde{Z}$ if and only if $\kappa^F = \kappa^D = 0$.*

Proof. The proof is standard and relies on the comparison of the first order conditions and resource constraints. It is available upon request. □

3.3.1 MRS vs. BKK

It should come as no surprise that our model approaches the BKK model as $\theta \to \infty$. Specifically, if an endogenous variable X is well defined in the BKK framework, we have:

$$\lim_{\theta \to \infty} X^{MRS(\theta)}\left(s^t\right) = X^{BKK}\left(s^t\right), \qquad \forall s^t.$$

3.3.2 FDI vs. domestic investment

The inter-temporal Euler conditions for domestically and foreign-located capital in the competitive equilibrium for household $i = A, B$ are as follows:

$$U_C^i\left(s^t\right) = \beta \sum_{s^{t+1}} \pi\left(s^{t+1}|s^t\right) \frac{\psi_i(s^{t+1})}{\psi_i(s^t)} U_C^i\left(s^{t+1}\right)\left[1 - \delta + MPK_i(s^{t+1}) \cdot \tilde{K}_{i,i}(s^t)\right]$$

$$U_C^i\left(s^t\right) = \beta \sum_{s^{t+1}} \pi\left(s^{t+1}|s^t\right) \frac{\psi_i(s^{t+1})}{\psi_i(s^t)} U_C^i\left(s^{t+1}\right)\left[1 - \delta + MPK_j(s^{t+1}) \cdot \tilde{K}_{j,i}(s^t)(1 - \kappa^F)\right]$$

where $U_C^i \equiv \frac{\partial U(C_i,L_i)}{\partial C_i}$ is the marginal utility of consumption, $MPK \equiv \alpha e^z \tilde{K}^{\alpha-1} L^{1-\alpha}$ is the marginal product of the effective capital stock, and $\tilde{K}_{i,j} \equiv \frac{\partial \tilde{K}_i}{\partial k_i^j}$ is the partial derivative of the effective capital stock $\tilde{K}_i$ w.r.t. to the capital owned by household j.

The most important price in our model is the return to capital. In country A, the rental rates on k_A and k_A^* are given by:

$$r_A = \alpha \exp(z) L_A^{1-\alpha} \left(\tilde{K}_A\right)^{\frac{1}{\theta}+\alpha-1} \cdot \omega_K \cdot k_A{}^{-\frac{1}{\theta}} \tag{3.11}$$

$$r_A^* = \alpha \exp(z) L_A^{1-\alpha} \left(\tilde{K}_A\right)^{\frac{1}{\theta}+\alpha-1} \cdot (1-\omega_K) \cdot k_A^*{}^{-\frac{1}{\theta}} \tag{3.12}$$

The equations above illustrate the important role that the elasticity of substitution plays in the decision to sell part of capital stock located in one country and purchase it in another country. The key is the impact of the changes in current level of effective capital stock, $\tilde{K}$, on the return to either k_A or k_A^*. When $\frac{1}{\theta} > 1-\alpha$, i.e. when the elasticity of substitution between domestic and foreign ownership is sufficiently small, a fall in $\tilde{K}$ reduces the rental rate. An outflow of foreign-owned capital, k_A^*, which reduces $\tilde{K}$ would then reduce r_A. This makes domestic residents reduce their holdings of domestic capital and, instead, purchase capital stock located in country B, generating a positive correlation between FDI inflows and outflows at a country level, and a positive co-movement of bilateral FDI flows.

3.3.3 Imperfect risk-sharing

A corollary to Theorem 3.2 is that perfect (up to preference shocks) risk-sharing, i.e. $\psi_A\left(s^t\right) U_C^A\left(s^t\right) = \psi_B\left(s^t\right) U_C^B\left(s^t\right), \forall s^t$, in the competitive equilibrium happens only if $\kappa^D = 0$. This follows from the direct comparison of the risk-sharing condition in the planner's problem and the inter-temporal

Euler conditions for debt in the competitive equilibrium below:

$$\begin{aligned}
&\text{(Planner)} && \psi_A(s^t)U_C^A(s^t) = \psi_B(s^t)U_C^B(s^t) \\
&\text{(Eq'm, country A)} && \psi_A(s^t)U_C^A(s^t) = \frac{1}{q(s^t)}\beta E\psi_A(s^{t+1})U_C^A(s^{t+1}) \cdot \frac{1}{1-\frac{\kappa^D}{1-\kappa^D}d(s^t)} \\
&\text{(Eq'm, country B)} && \psi_B(s^t)U_C^B(s^t) = \frac{1}{q(s^t)}\beta E\psi_B(s^{t+1})U_C^B(s^{t+1}) \cdot \frac{1}{1+\frac{\kappa^D}{1-\kappa^D}d(s^t)}.
\end{aligned}$$

4 Quantitative Analysis

We now investigate whether our augmented BKK framework, with imperfect substitution between domestic and foreign ownership of capital stock, can account for the positive co-movement of gross FDI flows. We calibrate the elasticity of substitution to ensure that in steady-state the model replicates the gross FDI positions observed in the data. Then, given the calibrated value of θ, we estimate the model using simulated method of moments. We then evaluate how well the model accounts for the positive co-movement of gross FDI flows, and how different parameters impact that co-movement (of course, we do NOT include the FDI co-movement in the set of targeted moments during the estimation).

4.1 Parameter values and functional forms

We impose values of a few parameters that are well established in the literature. The period in our model is one year, so we set the discount factor to $\beta = 0.96$; the depreciation rate of capital stock is set to $\delta = 0.05$; the capital share of national income is set to $\alpha = 0.33$. We consider three model specifications—with Cobb-Douglas, separable, or GHH preferences:

$$U(c,\ell) = \frac{\left[c^{1-\eta}\cdot(1-\ell)^{\eta}\right]^{1-\sigma}}{1-\sigma}, \quad \text{or} \quad U(c,\ell) = \frac{c^{1-\sigma}}{1-\sigma} - \eta\cdot\frac{\ell^{1+\gamma}}{1+\gamma}, \quad \text{or} \quad U(c,\ell) = \frac{\left[c-\eta\cdot\frac{\ell^{1+\gamma}}{1+\gamma}\right]^{1-\sigma}}{1-\sigma}.$$

In both cases we set the inter-temporal elasticity of substitution to $\frac{1}{\sigma} = \frac{1}{2}$, and we calibrate η so that in steady-state $\ell = 0.33$. In the specification with separable or with GHH preferences we set $\gamma = 2$ (Heathcote et al., 2008; Keane, 2011).

We jointly calibrate two parameters, θ and η, to match two steady-state moments. The elasticity of substitution between domestic and foreign ownership of the domestically located physical capital stock θ is calibrated by targeting the steady-state ratio of FDI liabilities over GDP $\frac{k_A^*}{y_A}$ (0.36 in the U.S. data). The weight put on hours worked in the utility function η is calibrated by targeting the steady-state level of ℓ (assumed to be 0.33).

The effective capital stock $\tilde{K}$ defined in (3.1) features two parameters—elasticity of substitution θ and home bias ω. Both will impact the $\frac{k_A^*}{y_A}$ ratio but we cannot identify the two parameters simultaneously. We therefore decided to set the benchmark value $\omega = 0.75$ following the empirical estimates in Ahearne et al. (2004) and then verify how sensitive our results are to this choice, by considering two alternative values - the high home bias with $\omega = 0.85$, and the low home bias with $\omega = 0.65$. Finally, we set the tax on FDI income outflow to be $\kappa^F = 0$.

4.1.1 Method of moments estimation

We compute the decision rules using first-order approximation around the non-stochastic steady-state. We then use Simulated Method of Moments (SMM) to estimate the model, using the empirical moments for the United States as targets.

Parameters There are nine parameters that we estimate: standard deviation and cross-country correlation of TFP shocks — σ_z and ρ_{z,z^*}; standard deviation and cross-country correlation of preference shocks — σ_ψ and ρ_{ψ,ψ^*}; the persistence of the TFP and preference shocks — ρ_z and ρ_ψ;

Table 6: Imposed and calibrated parameters

Parameter description	**Value**	**Sources**
Discount factor	$\beta = 0.96$	GJ, RS
Inter-temporal elasticity of substitution	$1/\sigma = 0.5$	HSV
Frisch elasticity of labor supply	$1/\gamma = 0.5$	HSV, K
Capital depreciation	$\delta = 0.05$	MR
Capital share	$\alpha = 0.3$	MR
FDI cost	$\kappa^F = 0.0$	n/a
Home Bias (benchmark)	$\omega = 0.75$	A
Elasticity of substitution — benchmark	$\theta = 2.12$	calibrated
— with $\omega = 0.85$	$\theta = 1.34$	calibrated
— with $\omega = 0.65$	$\theta = 3.76$	calibrated
Preference weight on hours worked — CD	$\eta = 0.62$	calibrated
— GHH	$\eta = 9.75$	calibrated
— SEP	$\eta = 50.73$	calibrated

GJ: Gourinchas and Jeanne (2013) RS: Rothert and Short (2022) HSV: Heathcote et al. (2008) K: Keane (2011) MR: Michaud and Rothert (2018) A: Ahearne et al. (2004)

and the portfolio adjustment costs κ^D.

Moments We target eleven moments in the estimation: standard deviation and persistence of real GDP; standard deviation (relative to that of GDP) and persistence of real consumption and investment expenditures; cross-country correlations of GDP, consumption, and investment expenditures; standard deviations (relative to that of GDP) of total gross FDI flows over GDP, and net exports over GDP. All moments are computed using residuals from the quadratic trend.

Estimation Since the model is over-identified, we follow the usual 2-step procedure in our estimation. Let $\mathbf{\Theta}$ be the vector of the estimated model parameters, let $\mathbf{m}^{model}(\mathbf{\Theta})$ be the vector of moments computed on the data simulated by the model (which, of course, will be a function of the parameter vector $\mathbf{\Theta}$), and let $\mathbf{m}^{data}$ be the same vector of empirical moments. In the first step we compute $\hat{\mathbf{\Theta}}_1$ as follows:

$$\hat{\mathbf{\Theta}}_1 := \arg\min_{\Theta} \| \mathbf{m}^{model}(\mathbf{\Theta}) - \mathbf{m}^{data} \|$$

In the second step, we first compute the variance-covariance matrix of the model generated moments, $\mathbf{V} \equiv Var\left(\mathbf{m}^{model}\left(\hat{\mathbf{\Theta}}_1\right)\right)$, and obtain the efficient method of moments estimator of $\mathbf{\Theta}$ to be:

$$\hat{\mathbf{\Theta}} := \arg\min_{\Theta} \left[\mathbf{m}^{model}(\mathbf{\Theta}) - \mathbf{m}^{data}\right]^T \cdot \mathbf{V}^{-1} \cdot \left[\mathbf{m}^{model}(\mathbf{\Theta}) - \mathbf{m}^{data}\right]$$

In the computation of the model generated moments, we simulate the model 10,000 times for 1540 periods, drop (burn) the first 1500 observations, compute 10,000 realizations of model-generated moments over the 40-year model observations (our data that we use to document the stylized facts in Section 2 spans 40 years), and take the average over those.

4.2 Results

The complete set of results is presented in Table 7. The model, in our view, does a remarkable job in accounting for a number of important non-targeted moments related to the behavior of the FDI flows. First and foremost, it generates a positive co-movement of gross FDI flows - something that has been a puzzle for the international business cycle literature (Bai, 2013). Additionally, it matches almost perfectly the correlations between real GDP and total FDI flows and between aggregate investment expenditure and lagged net FDI inflows. Moreover, each of the non-targeted correlations between FDI flows and either GDP or investment expenditure has the correct sign, including the positive co-movement of savings and investment (Feldstein and Horioka, 1980).

Table 7: Estimation Results

	Data	Model (s.e.)									
		Benchmark		$\omega = 0.85$		$\omega = 0.65$		GHH		SEP	
Targeted moments											
$\rho(y, y^*)$	0.458	0.435	(0.255)	0.452	(0.256)	0.447	(0.257)	0.479	(0.262)	0.472	(0.255)
$\rho(c, c^*)$	0.335	0.376	(0.267)	0.359	(0.278)	0.363	(0.276)	0.304	(0.292)	0.334	(0.280)
$\rho(x, x^*)$	0.397	0.415	(0.278)	0.442	(0.281)	0.445	(0.280)	0.404	(0.292)	0.423	(0.285)
$\rho(y_t, y_{t-1})$	0.789	0.612	(0.140)	0.621	(0.138)	0.619	(0.138)	0.638	(0.136)	0.625	(0.137)
$\rho(c_t, c_{t-1})$	0.868	0.611	(0.140)	0.623	(0.138)	0.621	(0.138)	0.625	(0.138)	0.618	(0.138)
$\rho(x_t, x_{t-1})$	0.699	0.628	(0.136)	0.638	(0.135)	0.638	(0.135)	0.640	(0.135)	0.637	(0.135)
$\sigma(y)$	3.117	3.181	(0.758)	3.181	(0.772)	3.181	(0.771)	3.163	(0.807)	3.177	(0.780)
$\sigma(c)/\sigma(y)$	1.128	1.083	(0.241)	1.110	(0.283)	1.117	(0.279)	1.219	(0.221)	1.137	(0.262)
$\sigma(x)/\sigma(y)$	2.815	2.811	(0.811)	2.815	(0.828)	2.816	(0.841)	2.822	(0.948)	2.816	(0.853)
$\sigma(nx/y)/\sigma(y)$	0.341	0.552	(0.154)	0.623	(0.186)	0.613	(0.182)	0.247	(0.079)	0.522	(0.158)
$\sigma(fdi)/\sigma(y)$	0.849	0.853	(0.250)	0.807	(0.243)	0.803	(0.246)	0.826	(0.273)	0.822	(0.251)
Non-targeted											
$\rho(fdi, fdi^*)$	0.595	0.336	(0.298)	0.478	(0.274)	0.199	(0.328)	0.286	(0.314)	0.292	(0.314)
$\rho(y, NETfdi)$	-0.616	-0.210	(0.321)	-0.202	(0.329)	-0.222	(0.325)	0.102	(0.341)	-0.139	(0.335)
$\rho(y, fdiIN)$	0.686	0.256	(0.294)	0.308	(0.294)	0.195	(0.311)	0.337	(0.299)	0.282	(0.301)
$\rho(y, fdiOUT)$	0.259	0.499	(0.251)	0.502	(0.255)	0.476	(0.267)	0.213	(0.317)	0.444	(0.274)
$\rho(y, fdiTOTAL)$	0.567	0.460	(0.248)	0.471	(0.254)	0.433	(0.264)	0.341	(0.289)	0.451	(0.260)
$\rho(x, fdiNET)$	-0.640	-0.469	(0.263)	-0.417	(0.288)	-0.440	(0.280)	-0.417	(0.285)	-0.444	(0.278)
$\rho(x, fdiIN)$	0.609	0.391	(0.284)	0.495	(0.268)	0.352	(0.303)	0.397	(0.293)	0.389	(0.294)
$\rho(x, fdiOUT)$	0.133	0.965	(0.027)	0.929	(0.054)	0.941	(0.046)	0.921	(0.059)	0.942	(0.045)
$\rho(x, fdiTOTAL)$	0.458	0.822	(0.120)	0.826	(0.123)	0.832	(0.119)	0.819	(0.128)	0.824	(0.123)
$\rho(x_t, fdiNET_{t-1})$	-0.367	-0.304	(0.274)	-0.273	(0.291)	-0.284	(0.286)	-0.279	(0.288)	-0.290	(0.284)
$\rho(x, s)$	0.447	0.664	(0.170)	0.664	(0.168)	0.665	(0.167)	0.683	(0.137)	0.730	(0.141)
Parameters											
σ_z		0.023	(0.000)	0.023	(0.000)	0.023	(0.000)	0.019	(0.000)	0.029	(0.000)
ρ_{z,z^*}		0.753	(0.005)	0.832	(0.006)	0.809	(0.006)	0.586	(0.004)	0.721	(0.005)
ρ_z		0.967	(0.003)	0.979	(0.002)	0.931	(0.002)	0.986	(0.003)	0.987	(0.002)
σ_ψ		0.146	(0.003)	0.196	(0.016)	0.213	(0.016)	0.215	(0.016)	0.185	(0.010)
ρ_{ψ,ψ^*}		0.170	(0.005)	0.173	(0.005)	0.202	(0.005)	0.155	(0.006)	0.145	(0.006)
ρ_ψ		0.898	(0.004)	0.952	(0.007)	0.835	(0.006)	0.943	(0.006)	0.934	(0.006)
ϕ_D		26.325	(0.485)	24.327	(0.471)	27.792	(0.551)	23.953	(1.464)	26.143	(0.791)
ϕ_F		87.950	(1.492)	99.939	(2.064)	99.603	(1.942)	87.447	(4.936)	94.166	(2.692)
κ^D		0.242	(0.007)	0.115	(0.008)	0.148	(0.010)	0.276	(0.015)	0.177	(0.009)

NOTES: Two-stage SMM estimates, based on 10,000 replications of the model over 1540 periods, with model moments computed over the last 40 periods.

Table 8: Counter-factual simulations

	Data	Benchmark Model	Counter-factuals $\theta = 10$	$\phi^D = 0$	$\phi^F = 0$	$\phi^D = \phi^F = 0$	BKK
Targeted moments							
$\rho(y, y^*)$	0.458	0.435	0.462	0.344	0.426	0.198	0.353
$\rho(c, c^*)$	0.335	0.376	0.373	0.237	0.275	0.223	0.236
$\rho(x, x^*)$	0.397	0.415	0.355	0.179	0.749	-0.321	0.161
$\rho(y_t, y_{t-1})$	0.789	0.612	0.613	0.653	0.602	0.676	0.654
$\rho(c_t, c_{t-1})$	0.868	0.611	0.612	0.626	0.601	0.621	0.627
$\rho(x_t, x_{t-1})$	0.699	0.628	0.630	0.554	0.532	0.237	0.558
$\sigma(y)$	3.117	3.181	3.175	5.407	3.874	5.805	5.459
$\sigma(c)/\sigma(y)$	1.128	1.083	1.082	1.047	1.068	0.989	1.047
$\sigma(x)/\sigma(y)$	2.815	2.811	3.035	8.716	5.482	11.137	8.865
$\sigma(nx/y)/\sigma(y)$	0.341	0.552	0.503	0.182	1.143	1.147	0.166
$\sigma(fdi)/\sigma(y)$	0.849	0.853	0.235	0.206	44.919	6.612	2.089
Non-targeted							
$\rho(fdi, fdi^*)$	0.595	0.336	0.115	-0.106	0.424	-0.923	-0.997
$\rho(y, NETfdi)$	-0.616	-0.210	-0.896	-0.314	-0.342	-0.058	-0.047
$\rho(y, fdiIN)$	0.686	0.256	0.192	0.241	0.209	0.058	-0.015
$\rho(y, fdiOUT)$	0.259	0.499	0.493	0.715	0.578	0.171	0.065
$\rho(y, fdiTOTAL)$	0.567	0.460	0.459	0.729	0.465	0.590	0.637
$\rho(x, fdiNET)$	-0.640	-0.469	-0.674	-0.328	0.307	0.636	0.109
$\rho(x, fdiIN)$	0.609	0.391	0.252	0.151	0.941	0.736	0.144
$\rho(x, fdiOUT)$	0.133	0.965	0.963	0.642	0.598	-0.514	-0.086
$\rho(x, fdiTOTAL)$	0.458	0.822	0.804	0.599	0.914	0.549	0.739
$\rho(x_t, fdiNET_{t-1})$	-0.367	-0.304	-0.423	-0.068	0.156	0.031	0.030
$\rho(x, s)$		0.664	0.719	0.918	0.482	0.639	0.929

NOTES: In the BKK model the gross flows are not well-defined. The last column corresponds to $\theta = 10$ and $\phi^D = \phi^F = 0$ and should be interpreted as the limit of our model as it approaches the BKK framework. All model-generated moments are averages from 10,000 simulations.

As long as capital is diverse there are two channels which help us account for the positive co-movement of FDI inflows and outflows. The first one is the less than perfect elasticity of substitution between domestic and foreign-owned capital stock — $\theta < \infty$. The second one is market segmentation, captured by the separate laws of motion for domestically and foreign-owned capital stocks combined with positive adjustment costs parameters, ϕ^D and ϕ^F. We now explore the role of these two channels separately and in conjunction. The results of our counter-factual simulations are presented in Figure 3 and in Table 8.

4.2.1 FDI co-movement vs. θ

Figure 3 shows how the correlation of gross FDI inflows and outflows changes as we change the values of θ. As θ approaches ten, which in the CES function is practically indistinguishable from infinity, the FDI co-movement drop from 0.34 to 0.11. In general, the negative relationship between θ and FDI co-movement is very strong. The intuition behind that relationship varies slightly depending on the source of exogenous shock, which we will now discuss.

Preference shocks Consider a one-time positive shock to the discount factor in country A. The shock has no direct impact on the return to capital, because it does not affect the marginal product of $\tilde{K}$. Since the households in country A are more patient, they want to save more, which means buying more capital stock. But the inter-temporal Euler equations imply that the country A households will be buying more of both domestic and foreign located capital. This means, we will see an increase in gross FDI outflows — $FDI_{A\to B}$ is rising. What about $FDI_{B\to A}$, i.e. gross FDI inflows?

The answer depends on the impact on r_A^* - the return to capital located in country A and owned by residents in country B. Since k_A is rising, $\tilde{K}_A$ is rising, and therefore $MPK \equiv \partial Y_A/\partial \tilde{K}_A$ is falling. However, $r_A^* = MPK \cdot \partial \tilde{K}_A/\partial k_A^*$. While an increase in k_A is lowering MPK, it will increase $\partial \tilde{K}_A/\partial k_A^*$, because k_A and k_A^* are not perfect substitutes. The greater the complementarity between k_A and k_A^*, the bigger will be positive impact of a rise in k_A on $\partial \tilde{K}_A/\partial k_A^*$. For low enough values of θ, the increase in $\partial \tilde{K}_A/\partial k_A^*$ will outweigh the decline in MPK, leading to an increase in r_A^*, which will then lead to an increase in k_A^*, i.e. an increase in $FDI_{B\to A}$ — gross FDI inflows into country A.

Productivity shocks Next, consider a positive productivity shock in country A. First, suppose that shock is transitory ($\rho_z = 0$). This means there is no impact on future productivity and the only impact is via an increase in today's income. Since K_A is held by households from both countries, we will see an increase in income in both countries. However, because of labor income, and because of home bias in $\tilde{K}$, the incomes of households in country A will increase more. households in country A want to save more. They start buying capital located in both countries, and we see an increase in gross FDI outflows — $FDI_{A \to B}$ is rising.

When the productivity shock is persistent — $\rho_z > 0$ — there is an additional effect: the future marginal product of the effective capital stock $\tilde{K}_A$ is rising. This has a direct effect on future returns to domestic and foreign ownership of local capital stock — it raises both $r_{A,t+1}$ and $r^*_{A,t+1}$. The increase in $r^*_{A,t+1}$ gives additional incentives to households in country B to buy capital located in country A, raising $FDI_{B \to A}$ (relative to the case of $\rho_z = 0$). This means that, for the same level of θ, we would expect a higher correlation between FDI inflows and outflows.

4.2.2 FDI co-movement vs. capital adjustment costs ϕ^D and ϕ^F

Figure 3 plots the gross FDI co-movement against the adjustment cost parameters (assuming $\phi^D = \phi^F$), with a reverse scale on the horizontal axis. As the adjustment costs approach zero, the FDI co-movement becomes strongly negative. However, around the values corresponding to the point estimates reported in Table 7, the relationship between gross FDI co-movement and adjustment costs is relatively flat and the FDI co-movement has a clear upper bound that is around 0.4.

The reason why positive adjustment costs on capital accumulation increase the gross FDI co-

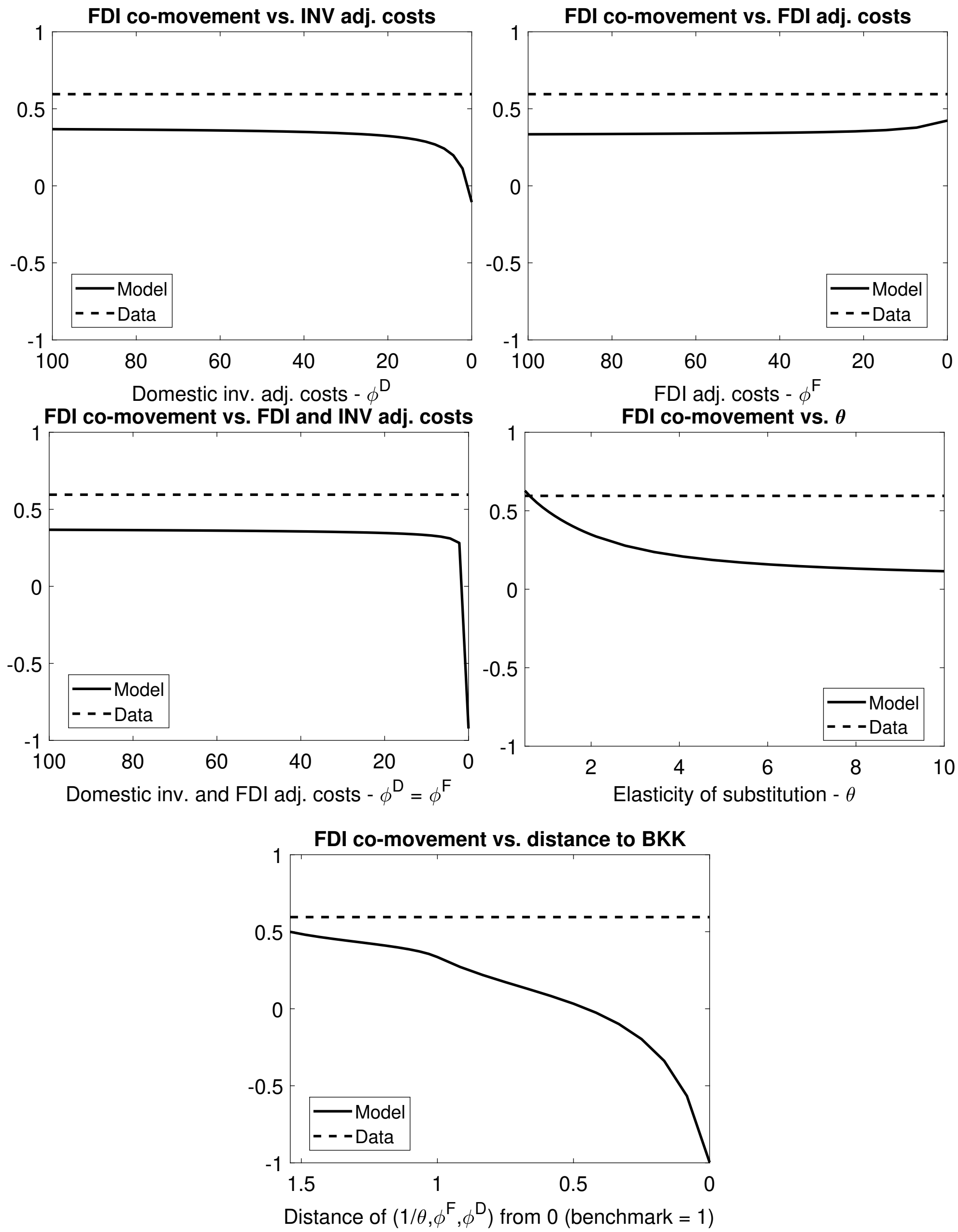


Figure 3: FDI co-movement: sensitivity to elasticity of substitution and investment adjustment costs (domestic and foreign)

movement in our model has to do with the cross-country correlation of productivity shocks that, in our benchmark estimation, is close to perfect. Consider a positive and persistent productivity shock in country A, that will typically coincide with an identical, persistent productivity shock in country B. A household in country A starts purchasing more capital stock located domestically. However, the convex adjustment costs start kicking in very quickly, and thus the country A household has a strong incentive to also start buying the capital stock located in country B, hence $FDI_{A\to B} \uparrow$. A household in country B is facing identical incentives, hence $FDI_{B\to A} \uparrow$.

4.2.3 FDI co-movement vs. distance from BKK

Our model approaches the BKK model as $\theta \to \infty$, $\phi^D \to 0$, and $\phi^F \to 0$. While the gross FDI flows are not defined in the BKK framework, it is worth looking at their behavior as our model approaches that limit. This is depicted in the right-most panel in Figure 3 and in the last column of Table 8 (corresponding to $\theta = 10$, and $\phi^D = \phi^F = 0$). The message is clear: as the model gets closer to the BKK framework, the correlation between gross FDI flows approaches negative one.

Our earlier discussion indicates that, in order to account for a positive co-movement of gross FDI flows, it is necessary to model the accumulation of domestic and foreign owned capital stocks separately, and introduce adjustment costs in the two laws of motion. At the same time, however, a sufficiently low value of the elasticity of substitution is necessary to bring the FDI co-movement in the model reasonably close to the empirical estimate of 0.59.

4.2.4 FDI openness, long-run welfare effects, and measured TFP

The value of θ, calibrated to match the ratio of gross FDI liabilities to GDP in the United States, was 2.12. The model with that value of θ did a very good job in accounting for the positive co-movement of gross FDI flows, as well as in matching a number of other non-targeted correlations of FDI flows with GDP and with aggregate investment expenditures. As such, we interpret our results as evidence in favor of less than perfect substitution between domestically and foreign-owned capital stock. Such *capital diversity* offers a new source of welfare gains from openness to FDI - a more efficient allocation (across domestic and foreign firms) of domestically located physical capital stock.

Our measure of long-run gains from FDI openness is the percentage change in the steady-state level of consumption associate from reducing κ^F from 0.9 to 0.1:

$$\Delta W \equiv \log\left(c\left(\kappa^F = 0\right)\right) - \log\left(c\left(\kappa^F = 0.9\right)\right)$$

With $\theta = 2.12$, the welfare gains from FDI openness stemming from the *capital diversity* channel are equivalent to 4.2 % increase in steady-state consumption in the case of the Cobb-Douglas utility, 3.1% increase in the case of separable preferences, and 6.3% in the case of GHH preferences. It is important to emphasize that those welfare gains are orthogonal to the ones from risk sharing or from eliminating capital scarcity. In that sense, the new *capital diversity* channel suggests that total welfare gains from financial openness far exceed the elusive gains calculated in earlier studies (Gourinchas and Jeanne, 2006).

Steady-state consumption in the model is higher in the world with free movement of FDI because capital is being more efficiently allocated. In the data, however, if empiricist ignored effective capital stock (as we have modelled it) and simply assumed foreign and domestic capital were perfect substitutes the gains from FDI would manifests themselves as an increase in measured total factor productivity (the Solow Residual). And indeed a large body of empirical evidence present in the literature find that capital account openness is associated with an increase in measured total factor productivity of domestic firms (Smarzynska Javorcik, 2004; Li and Su, 2022).

We are able to endogenize the impact of FDI openness on measured TFP, parsing it out from the Solow Residual. In the model, we can define the steady-state level of measured TFP, as a function of barriers to FDI κ^F, as follows:

$$TFP(\kappa^F) \equiv \frac{Y(\kappa^F)}{K(\kappa^F)^{\alpha}\ell(\kappa^F)^{1-\alpha}} = e^{\bar{z}}\left[\frac{\tilde{K}(\kappa^F)}{K(\kappa^F)}\right]^{\alpha}$$

where $K(\kappa^F) \equiv k(\kappa^F) + k^*(\kappa^F)$ and $\tilde{K}(\kappa^F) \equiv \left[\omega k(\kappa^F)^{\frac{\theta-1}{\theta}} + (1-\omega)k^*(\kappa^F)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$. The impact that changes in κ^F have on TFP is summarized in Proposition 4.1, which states that an increase in the barriers to FDI reduces TFP, and that the associated decrease in TFP is larger when the elasticity of substitution between the domestic and foreign ownership of the aggregate capital stock is smaller.

Proposition 4.1. *Let $TFP(\kappa^F;\theta)$ be the steady-state level of the Solow Residual, given the barriers to FDI κ^F and the elasticity of substitution θ. Then*

$$0 > \frac{\partial TFP(\kappa^F;\theta_1)}{\partial \kappa^F} \geq \frac{\partial TFP(\kappa^F;\theta_2)}{\partial \kappa^F} \text{ if and only if } \theta_2 \leq \theta_1 < \infty$$

Proof. First, notice that $K(\kappa^F, \theta)$ required to produce one unit of $\tilde{K}$ is given by the value attained in the following minimization problem:

$$K(\kappa^F, \theta) \equiv \min_{k,k^*} k + k^*$$

subject to: $1 \leq \left[\omega k^{\frac{\theta-1}{\theta}} + (1-\omega)k^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$ and $1-\kappa^F = \frac{1-\omega}{\omega}\left(\frac{k}{k^*}\right)^{\frac{1}{\theta}}$. It is then straightforward to show that $\frac{\partial K}{\partial \kappa^F} > 0$ and that this derivative is decreasing in θ. The proof is available upon request. $\square$

5 Conclusions

In this paper we offered a very flexible framework to study gross FDI flows over the international business cycle, opening doors for a research agenda focusing on that important aspect of open economy macroeconomics. We allowed for the possibility that domestic and foreign ownership of capital stock were not perfect substitutes. This small deviation from the standard IBC model allowed us to capture gross FDI flows within the classic BKK framework. We showed that the standard BKK model (a limiting case of our economy) delivers a strong counter-factual prediction of a perfectly negative correlation of FDI inflows and outflows, that is reversed when the elasticity of substitution between domestic and foreign ownership of capital is sufficiently low.

Imperfect substitution between domestic and foreign ownership of capital means that financial openness can improve welfare via a new channel that we dubbed "capital diversity". Our results suggest this channel is quantitatively important—openness to FDI yields welfare gains equivalent to about 5-6% increase in lifetime consumption. Importantly, those gains are orthogonal to any additional gains stemming from risk-sharing or consumption smoothing. This might call for potential re-evaluation or fine-tuning of some important, and recently popular arguments favoring so-called capital controls (Jeanne and Korinek, 2010; Bianchi, 2011; Costinot et al., 2014; Michaud and Rothert, 2014).

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6 Appendix

Table 9: Business Cycle Statistics

	G7	USA	Germany	France	Japan	Canada	Italy	GB
sdev (Y)	2.826	3.117	2.429	2.334	3.579	1.942	2.451	3.932
sdev(C) / sdev (Y)	1.181	1.128	1.122	1.044	0.661	1.901	1.034	1.378
sdev (I) / sdev (Y)	3.457	2.815	3.442	3.080	2.892	5.146	4.050	2.776
sdev (NX) / sdev (Y)	0.523	0.341	0.554	0.495	0.308	0.931	0.677	0.358
sdev (Gross FDI) / sdev (Y)	0.838	0.343	1.064	1.086	0.157	1.245	0.567	1.400
sdev (Net FDI) / sdev (Y)	0.450	0.213	0.600	0.652	0.144	0.631	0.264	0.645
sdev (Gross Portfolio) / sdev (Y)	1.453	0.680	1.604	2.469	0.587	1.135	2.009	1.688
sdev (Net Portfolio) / sdev (Y)	1.032	0.454	0.988	1.384	0.717	1.263	0.940	1.478
sdev (Gross FDI and Portfolio) / sdev (Y)	1.839	0.849	2.296	3.168	0.600	1.471	2.104	2.383
sdev (Net FDI and Portfolio) / sdev (Y)	1.002	0.449	0.986	1.472	0.732	1.084	0.903	1.386
ρ (C, Y)	0.824	0.957	0.854	0.844	0.880	0.410	0.853	0.967
ρ (I, Y)	0.801	0.855	0.847	0.825	0.928	0.414	0.852	0.883
ρ (NX, Y)	-0.231	-0.603	-0.068	-0.105	0.144	0.091	-0.442	-0.636
ρ (FDI In, FDI Out)	0.547	0.453	0.595	0.670	0.124	0.648	0.658	0.684
ρ (Portfolio In, Portfolio Out)	0.272	0.450	0.463	0.528	-0.196	-0.119	0.641	0.137
ρ (FDI and Portfolio In, FDI and Portfolio Out)	0.466	0.595	0.696	0.686	-0.198	0.297	0.692	0.497
ρ (Gross FDI, Y)	0.460	0.477	0.230	0.588	0.286	0.501	0.576	0.560
ρr (Net FDI, Y)	0.066	-0.016	-0.060	0.351	0.326	-0.398	0.100	0.158
ρ (FDI Out, Y)	0.401	0.364	0.240	0.523	0.331	0.333	0.512	0.502
ρ (FDI In, Y)	0.405	0.460	0.188	0.603	-0.016	0.525	0.541	0.534
ρ (Gross Portfolio, Y)	0.155	0.467	0.064	0.382	0.196	-0.451	0.073	0.351
ρ (Net Portfolio, Y)	-0.074	-0.602	-0.153	0.102	-0.055	0.523	-0.113	-0.219
ρ (Portfolio Out, Y)	0.104	0.078	-0.030	0.356	0.081	0.117	0.019	0.106
ρ (Portfolio In, Y)	0.169	0.586	0.121	0.309	0.170	-0.596	0.113	0.481
ρ (Gross FDI and Portfolio, Y)	0.337	0.567	0.151	0.499	0.267	0.069	0.225	0.578
ρ (Net FDI and Portfolio, Y)	-0.060	-0.616	-0.190	0.251	0.010	0.370	-0.088	-0.160
ρ (FDI and Portfolio Out, Y)	0.259	0.259	0.069	0.483	0.165	0.252	0.182	0.400
ρ (FDI and Portfolio In, Y)	0.310	0.686	0.200	0.427	0.175	-0.161	0.230	0.610

Note: All statistics calculated after taking a quadratic filter of each series. For alternative filtering processes, see data appendix.